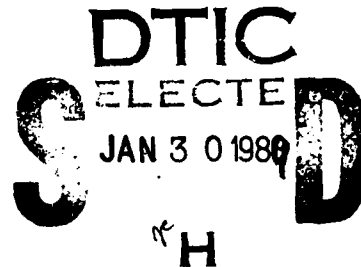④

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A ROBUST ADAPTIVE ARRAY STRUCTURE USING THE SOFT CONSTRAINED LMS ALGORITHM

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ABSTRACT

This paper presents a new algorithm for adaptive beamforming which is robust to both desired signal misalignment and array geometry imperfections. It extends the work done by Jablon and Widrow [1] by utilizing *a priori* knowledge of the desired signal direction to generate new soft constraints on the pattern of the array.

1. INTRODUCTION

Adaptive beamformers can be used, with minimal a priori information, to greatly improve the signal-to-noise ratio of a signal in the prescence of hostile interference (jammers). However, when the actual signal angle of arrival does not coincide with the estimated angle of arrival, or when the actual geometry of the array differs from the geometry assumed by the array designer, then the perfcrmance of the array may be degraded due to adaptation. This phenomena is due to the fact that in the absence of a pilot signal, the usual adaptive array attempts to minimize total output power, regardless of the type of signal present. While this is desirable if the received signal is mostly jammer and receiver noise, when the desired signal component becomes large, the receiver attempts to reject it, thus significantly degrading performance [2,3].

The most common way of avoiding the problem of rejecting the signal along with the noise is to impose some sort of linear constraints on the ways in which the array is allowed to adapt. Many methods have been proposed to implement these types of algorithms, but the most common are the Frost algorithm and the Generalized Sidelobe Canceller (GSC) structure [4,5]. Both of these types of arrays impose constraints on the array patterns which they may generate, usually either fixed direction gain constraints, or constraints on the derivative of gain vs. angle at a fixed look direction.

The array processor proposed in this study is designed to give the maximum number of degrees of freedom for the processor to null out interferers, while being very robust with respect to both array geometry and signal direction. This is achieved by use of the available *a priori* knowledge of the signal direction to generate soft constraints on the processor weights.

2. SYSTEM MODEL.

For the remainder of the paper, the structure of the adaptive array under consideration will be that of a zeroth order GSC. This structure has been shown to converge to the same steady state solution as the Frost constrained adaptation algorithm with one directional constraint. We will be interested in the case of narrowband systems, with sources emitting only plane wave signals (i.e. far field approximation). We will assume one jammer with unknown direction, and a signal whose direction is random, but not uniformly distributed over the entire $(-\pi, \pi)$ angle. We also assume that the antenna array is linear, with uniform $\lambda/2$ spacing between antennas. However, the gain of each antenna is assumed to be a complex gaussian random variable with inphase and quadrature parts both distributed as $N(1, \sigma^2_g)$, all uncorrelated with each other. This model is also appropriate for an array where the antenna placements are

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random, but close to linear.

It should be noted that the assumptions of a uniform linear array and one jammer are made purely for mathematical expediency, and are not necessary for the results of this paper to hold.

The sidelobe cancellation matrix $\mathbf{B}$ is designed to be an N x N-1 matrix of rank N-1, such that

$$\mathbf{B}\vec{s}=\vec{0} \tag{1}$$

where $\vec{s}$ is the vector of delays seen for a plane wave signal coming from the constraint direction. Normally, the array is assumed to be *pre−steered* so that the constraint direction is broadside to the array, and thus :

$$\vec{s}=\vec{1} \tag{2}$$

The output of the GSC, using our assumptions on the gain of the antenna elements, is given by the expression:

$$\vec{1}'\mathbf{G}\vec{x}-\vec{w}'\mathbf{B}\mathbf{G}\vec{x} \tag{3}$$

where $\vec{x}$ is the received vector, and $\vec{w}$ is the N-1 dimensional vector of weights used in weighting the outputs of the $\mathbf{B}$ matrix.

$\mathbf{G}$ is the diagonal matrix of the antenna gains, and can be written as:

$$\mathbf{G}=diag(1+\Delta g_1, 1+\Delta g_2, ... 1+\Delta g_N)=\mathbf{I}+\Delta\mathbf{G}$$

and Δg_i is a $N(0,\sigma^2_g)$ complex random vector.

We now define the output of the beamforming matrix as $\vec{u}$, and look at the optimization problem of minimizing the total output power of the GSC, which translates to the problem:

$$\text{MIN}_{\vec{w}} \quad E[\,|\,\vec{1}'\mathbf{G}\vec{x}-\vec{w}'\vec{u}\,|^2] \tag{4}$$

The solution to this problem is the well known Weiner Filter solution. The optimum weight vector $\vec{w}$ is given by the equation:

$$\vec{w}=\mathbf{R}_{uu}^{-1}\vec{r}_{ud} \tag{5}$$

If we look at the case of a single jammer, a desired signal, and uncorrelated receiver noise, then the matrix $\mathbf{R}_{uu}$ looks like:

$$\mathbf{R}_{uu}=\mathbf{B}\,[\,\sigma^2_n\mathbf{I}+\mathbf{G}\mathbf{R}_{ss}\mathbf{G}^\dagger+\sigma^2_j\mathbf{G}\vec{j}\vec{j}^\dagger\mathbf{G}^\dagger\,]\mathbf{B}^\dagger \tag{6}$$

Where the vectors $\vec{s}$ and $\vec{j}$ are vectors of delays seen at the antennas for plane wave signals coming from the desired signal and jamming directions respectively. The matrix $\mathbf{R}_{ss}$ is defined as:

$$\mathbf{R}_{ss}=\sigma^2_s E[\vec{s}\vec{s}^\dagger] \tag{7}$$

And the vector $\vec{r}_{ud}$ is of the form:

$$\vec{r}_{ud}=\mathbf{B}\mathbf{G}\mathbf{R}_{zz}^\dagger\mathbf{G}^\dagger\vec{1} \tag{8}$$

$$\text{Where } \mathbf{R}_{zz}=E[\vec{x}\vec{x}^\dagger]$$

And $\vec{x}^\dagger$ and $\vec{x}^*$ denote complex conjugate transpose and complex conjugate of $\vec{x}$ respectively.

3. ROBUST ADAPTATION ALGORITHM

If the signal direction vector $\vec{s}$ is not exactly equal to $\vec{1}$, or if the variance of the antenna gains is not very small, the array pattern generated by (4) will attempt to null out a strong

desired signal. To deal with the problem of random antenna gains, Jablon and Widrow [1] have proposed using a soft constraint type of optimization. A suitable penalty function is chosen, and the optimization problem is then changed to minimizing the original functional, (in this case $E[\mid \vec{1}'G\vec{x} - \vec{w}'\vec{u} \mid^2]$) plus some penalty constant times the penalty function. If the penalty constant is chosen to be large enough, then the optimization problem will have a solution very close to that of a hard constraint problem, where the constraint is defined as the penalty function being forced to take on the value zero.

The penalty function chosen in [1] attempted to minimize the leakage of the signal component into the cancellation branch due to random antenna gains. Here, we extend this technique to the case of both unknown antenna gains and unknown signal directions, and derive a new LMS algorithm for this case. We then show how the results can be extended to give a reasonably efficient algorithm by choosing the correct type of beamforming matrix.

We will define our penalty function $p(\vec{w})$ as one which is linear in the amount of power that is transmitted through the cancellation branch of the GSC due to the signal component. The leakage of the signal vector into the cancellation branch is given by the expression:

$$L(\vec{s}) = \vec{w}'BG\vec{s} \tag{9}$$

We will take the expected value of the power of the leakage signal as our penalty function. Thus, $p(\vec{w})$ becomes:

$$p(\vec{w}) = E[\vec{w}'B(G\vec{s}\vec{s}^{\dagger}G^{\dagger})B^{\dagger}\vec{w}^{*}] \tag{10}$$

Bringing the ensemble expectation inside and utilizing the assumptions that the errors in the antenna gains are independent of each other, and of the possible signal directions, and also that the variance of the components of the signal vector are all equal to some constant, σ^2_{ss} , we get the simplified result:

$$p(\vec{w}) = \vec{w}'B(\sigma^2_{g}\sigma^2_{ss}I + R_{ss})B^{\dagger}\vec{w}^{*} \tag{11}$$

And now the soft constrained optimization problem becomes:

$$\text{MIN}_{\vec{w}} \quad \vec{1}'R_{zz}\vec{1} - 2\text{Re}[\vec{w}'BR_{zz}\vec{1}] + \vec{w}'B[R_{zz} + c(\sigma^2_{g}\sigma^2_{ss}I + R_{ss})]B^{\dagger}\vec{w}^{*} \tag{12}$$

At this point, we note that the term is quadratic in the vector $\vec{w}$, and thus any minima of this expression will be global. The matrix R_{ss} is the spatial autocorrelation matrix for the desired signal. This matrix is assumed to be known *a priori*, but as we will show later, any reasonable estimate will suffice for the proposed algorithm.

We now take the instantaneous approximation to the gradient vector of our error surface using the usual LMS methods and subtract some multiple of it from our present solution for $\vec{w}$. This method gives us a new adaptive algorithm for the tap weights:

$$\vec{w}_{k+1} = (I - 2\mu(\vec{u}^{*}\vec{u}^{t} + \frac{\eta}{2\mu}B[\sigma^2_{ss}\sigma^2_{g}I + R_{ss}]B^{\dagger})\vec{w}_{k} + 2\mu\vec{x}'\vec{1}B\vec{x}^{*} \tag{13}$$

Or, after re-arranging terms:

$$\vec{w}_{k+1} = [I - \eta\sigma^2_{ss}\sigma^2_{g}BB^{\dagger} - \eta BR_{ss}B^{\dagger}]\vec{w}_{k} + 2\mu y_{k}\vec{u}^{*}_{k} \tag{14}$$

Where the term y_{k} is the output of the array at the kth iteration. This algorithm can be shown to converge for positive values of μ such that:

$$\mu < \frac{1}{tr[R_{uu} + \frac{\eta}{2\mu}B(\sigma^2_{g}\sigma^2_{ss}I + R_{ss})B^{\dagger}]} \tag{15}$$

It should be noted that the matrix $BR_{ss}B^{\dagger}$ is Toeplitz, non-negative definite, and hermitian. Thus, the matrix is diagonalizable by a unitary matrix. Let this matrix be denoted by P. Then, we can write (14) as:

$$\overrightarrow{\mathbf{w}}_{k+1} = [\,\mathbf{I} - \eta \sigma^2_{ss}\sigma^2_g \mathbf{B}\mathbf{B}^\dagger - \eta \mathbf{P}\Lambda_{ss}\mathbf{P}^\dagger\,]\overrightarrow{\mathbf{w}}_k + 2\mu y_k \overrightarrow{\mathbf{u}}^{\,*}_k \qquad (16)$$

Where the matrix Λ_{ss} is the diagonal matrix of eigenvalues of the matrix $\mathbf{B}\mathbf{R}_{ss}\mathbf{B}^\dagger$. We now change coordinates by using the operator $\mathbf{P}$, and define a new beamforming matrix $\overline{\mathbf{B}}$ by the expression:

$$\overline{\mathbf{B}} = \mathbf{P}\mathbf{B} \qquad (17)$$

The matrix $\overline{\mathbf{B}}$ has the required property of a beamforming matrix, i.e. equation (1) is true for $\overline{\mathbf{B}}$ if it is true for the matrix $\mathbf{B}$. Using this new beamforming matrix, the update equations now become:

$$\overrightarrow{\mathbf{w}}_{k+1} = [\,\mathbf{I} - \eta\sigma^2_{ss}\sigma^2_g \overline{\mathbf{B}}\,\overline{\mathbf{B}}^\dagger - \eta\Lambda_{ss}\,]\overrightarrow{\mathbf{w}}_k + 2\mu y_k \overrightarrow{\mathbf{u}}^{\,*}_k \qquad (18)$$

Where the vectors $\overrightarrow{\mathbf{w}}$ and $\overrightarrow{\mathbf{u}}$ are defined in the new coordinate system.

4. RESULTS

The new algorithm requires the knowledge of the signal spatial autocorrelation matrix $\mathbf{R}_{ss}$ a priori. In practice, this can be approximated by assuming that the signal will definitely fall within the main beam of the array (or else the array would be mis-steered), and then taking the maximum variance distribution within this arc. Using this method, the spatial distribution of the desired signal is assumed constant within some arc symmetric about broadside, and zero elsewhere. Since this is an even symmetric function, the matrix $\mathbf{R}_{ss}$ has all real entries.

The quiescent pattern generated by the new adaptation algorithm was calculated analytically for the case of an 11 element linear array in the presence of one signal and one jammer The SINR was calculated as a function of jammer angle of arrival for a fixed signal location, and then again as a function of signal angle of arrival with a fixed jammer location. The results show that the loss in SINR due to lack of exact knowledge of the desired signal direction is very small as long as the jammer lies outside the main lobe of the array pattern, and the actual signal vector lies inside the main lobe. This is due to the fact that the soft constraint "freezes" the pattern of the array in the main lobe region, while allowing it to adapt in other regions.

A comparison of the output SINR of the new soft constrained algorithm and a first order constrained GSC shows that the new algorithm is much less sensitive to errors in desired signal direction of arrival. In addition, trade-offs between ability to null jammers close to broadside and robustness to signal misalignment can be made through changes in $\mathbf{R}_{ss}$.

5. CONCLUSION

This paper develops a new algorithm for robust beamforming in the prescence of uncertain signal directions and array geometries. The new algorithm incorporates available a priori information into a penalty function which imposes soft constraints on the adaptation of the array. The proposed algorithm is shown to be better than linearly constrained adaptive arrays at handling off center desired signals.

[1] N. K. Jablon, "Adaptive Beamforming with The Generalized Sidelobe Canceller in the Presence of Array Imperfections," *IEEE Trans. Ant. Prop.*, vol. AP-34, no. 8, pp. 996-1012, Aug. 1986.

[2] C. L. Zahm, "Effects of Errors in the Direction of Incidence on the Performance of an Adaptive Array," *Proc. IEEE*, vol. 60, no. 8, pp. 1008-1009, Aug. 1972.

[3] A. M. Vural, "Effects of perturbations on the performance of optimum/adaptive arrays," *IEEE Trans. Aerosp. Electron. Syst.*, vol. AES-15, no. 1, pp. 76-87, Jan. 1979.

[4] O. L. Frost, III, "An algorithm for linearly constrained adaptive array processing," *Proc. IEEE*, vol. 60, no. 8, pp. 926-935, Aug. 1972.

[5] L. J. Griffiths and C. W. Jim, "An Alternative approach to Linearly Constrained Adaptive Beamforming," *IEEE Trans. Ant. Prop.*, vol. AP-30, no. 1, pp. 27-34, Jan. 1982.
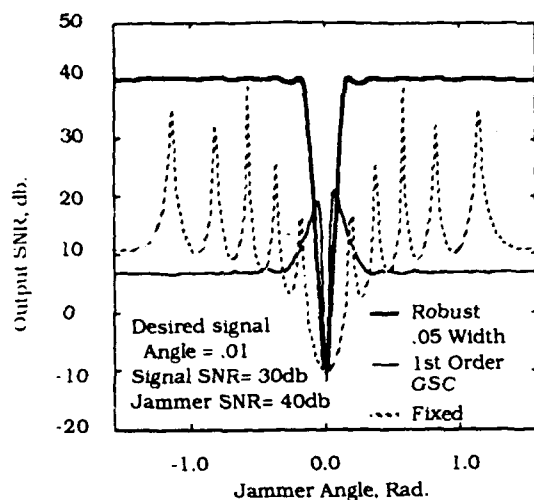
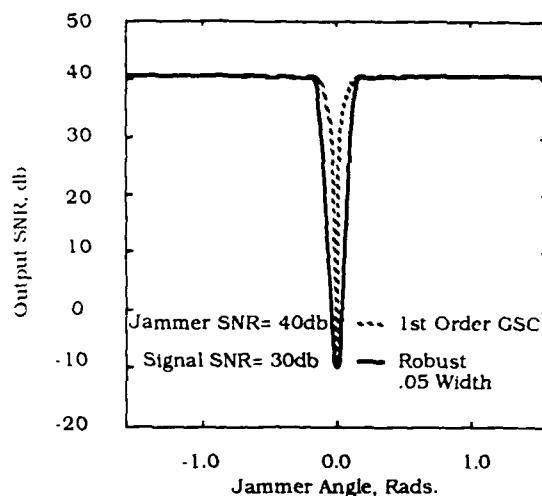Fig. 1: Output SNR vs. Jammer Angle

Fig. 2: Output SNR vs. Jammer Angle. No Misalignment

Results for a linear 11 element array
Width of signal spatial autocorrelation
matrix for robust algorithm is in radians.

Fig. 1-3: Output SNR for adaptive processors
using infinitely slow adaptation.

Fig. 4-5: Array patterns due to simulated gaussian
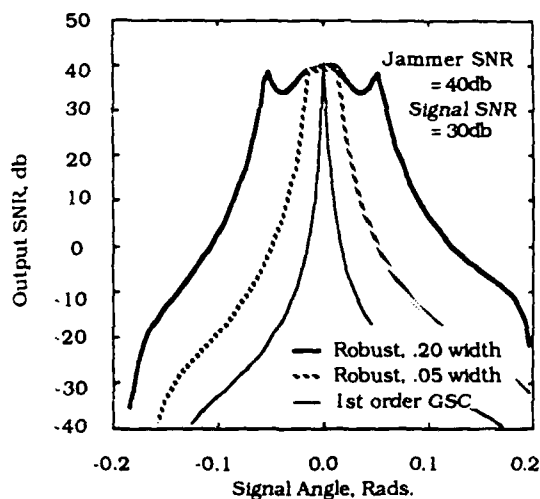noise, jammer, and signal environment.

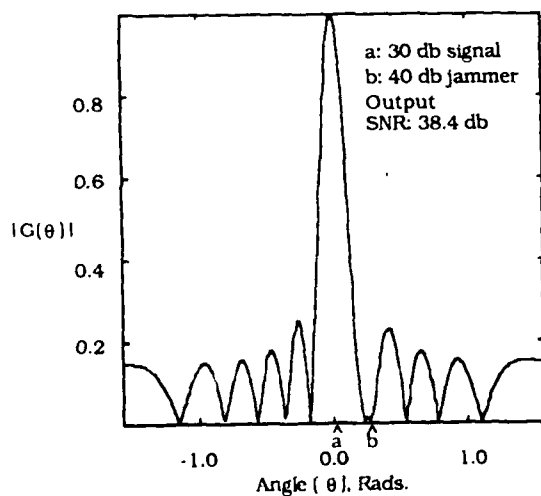Fig. 3: Output SNR vs. Signal Angle, Jammer at .27 Rads.

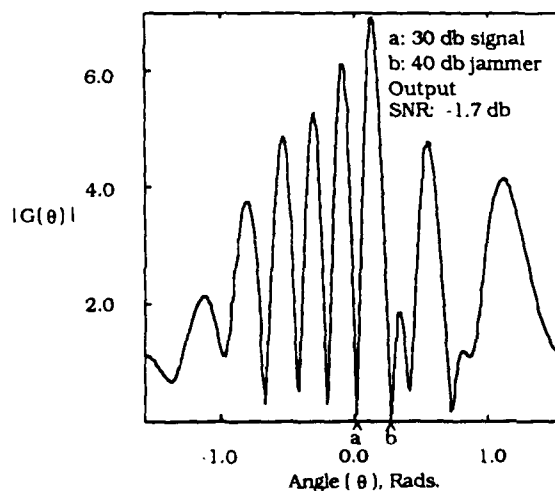Fig. 4: Simulated pattern due to Robust algorithm

Fig. 5: Simulated pattern due to 0th order GSC.

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